

Mobius3D White Paper: Dose Calculation Algorithm

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Introduction

Mobius3D uses a new, independently developed collapsed cone (CC) convolution/superposition (CS) algorithm to calculate photon dose distributions.

Mobius3D's dose calculations are conceptually similar to the CC calculations performed in Philips Pinnacle3 software. Both are rooted in

the principles of the original collapsed cone research publications from the 1980s including works by Mohan, Ahnesjo, and Mackie¹⁻⁴. Both have been updated with enhancements based on subsequent research⁵⁻⁹. In general, Mobius3D's collapsed cone algorithm performs a more rigorous calculation than Pinnacle by making fewer assumptions.

	Pinnacle 3 (CCC setting)	Mobius3D
Fluence: Gaussian source spread	Yes	Yes
Fluence: MLC rounded leaf ends	Yes	Yes
Fluence: MLC tongue and groove	Yes	Yes
Fluence: Arbitrary intensity profiles	Yes	Yes
TERMA: Material-dependent attenuation	Yes	Yes
TERMA: Calculations for every voxel	Yes	Yes
TERMA: Energy-dependent attenuation	Yes	Yes
Electron contamination	Yes	Yes
Automatic beam modeling	No	Yes
Collapsed cone: Number of cones	80	144
Collapsed cone: Calculation step size always ≤ 1 voxel	No	Yes
Collapsed cone: High resolution kernels	No	Yes
Collapsed cone: Number of polyenergetic kernels per calculation	1	1
Dose grid automatically defined based on beam size	No	Yes
Approximate calculation time per beam	40 s	1 s

Collapsed Cone Modeling Principles

All CCCS models share the same common principles for calculating dose for a radiation field. They use input data from a beam model which represents the radiation delivery characteristics of a specific machine / energy combination. Next, they calculate primary 2D energy fluence from the linear accelerator one field at a time. Then the fluence energy is projected through the patient and a 3D TERMA distribution is calculated. Finally, the TERMA is convolved with a dose spread function to calculate 3D dose deposition.

Beam Modeling

Beam modeling is necessary to represent the characteristics of the radiation delivered by a specific machine and nominal energy combination. Some of the key characteristics include:

- Energy spectrum
- Output factors
- Profile definition
- Geometric machine characteristics

The beam model forms the foundation for every dose calculation step.

Fluence

The amount of photon energy radiating directly from the linear accelerator per unit area is known as primary energy fluence. Primary energy fluence is calculated in a reference plane perpendicular to the beam, temporarily ignoring any energy attenuation caused by photon interactions with the patient. Fluence is not constant across the reference plane because of collimation patterns of the field and

other physical machine characteristics represented by the beam model.

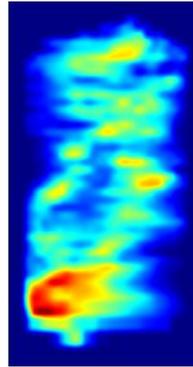


Figure 1: Fluence from an MLC-modulated treatment field.

TERMA - Total Energy Released in Matter

When a photon beam radiates through material (i.e. a patient), the photons collide with molecules of the patient. This transfers some of the primary fluence energy from the beam to the patient. A 3D TERMA distribution represents how much energy is released in the patient and where it is released (note this is not the same as where the energy is ultimately deposited).

The mathematical model for TERMA is based on the product of the primary energy fluence and the mass attenuation coefficient of the material at the point of interest. Taking into account that the primary energy fluence is attenuating downstream because of photon collisions, this determines a model for TERMA very close to exponential decay. The full TERMA model deviates somewhat from exponential decay due to variable patient density and the point source-like beam divergence.

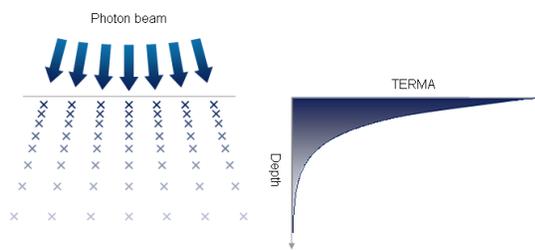


Figure 2: Illustration of a photon beam hitting a flat surface and creating exponentially-decaying TERMA.

Dose Deposition

Convolution/Superposition

Dose is energy deposited per mass. In a CS construction, the TERMA distribution is convolved with a dose-spread function to determine a dose distribution. As described earlier, TERMA defines how much energy has been released, but that energy can be deposited close to or far away from where it was released. Photoelectric interactions result in electrons depositing dose within a few mm, but Compton interactions release secondary photons that can travel a range of distances before depositing their freed energy. Therefore, since the electron displacement and energy deposition is chaotic, dose spread functions (kernels) are developed using Monte Carlo simulations. The summing of the calculated dose distributions from all points where energy is released (superposition) determines the ultimate overall dose distribution.

It is important to note in CS that the Monte Carlo-derived dose spread functions are typically calculated in a water phantom, whereas patients have tissue, bone, lung, etc. Thus, CS heterogeneity corrections depend on density (ray tracing algorithms use density to calculate radiological distance), but do not correct for the different atomic compositions of patients. In practice, for high energy photons, lung, tissue, fat, and even bone interact in a

similar manner as water. However, low-energy photons in bone are more readily absorbed due to the photoelectric effect, and high Z materials (like Titanium) can have significantly different interactions than water-based dose spread functions would predict.

Despite using material-independent dose spread functions calculated in a sphere of water (versus the exact geometry of the patient at each TERMA voxel), CS calculations are typically very accurate in patients. Ray tracing accounts for density differences, and the dose spread function is able to account for heterogeneities and patient geometry in 3D.

Why Use Collapsed Cone?

A full CS model is one where the dose deposition is specifically calculated from every voxel in the grid to every other voxel in the grid. This would require approximately 100^7 (one hundred trillion) calculations for a 100^3 dose grid (ie, a 30x30x30 cm grid with 3 mm resolution). This is impractical with today's computers, so instead a CC algorithm distributes dose along a discrete set of rays (the centers of energy cones) that emanate from each TERMA point, reducing the calculation time by several orders of magnitude.

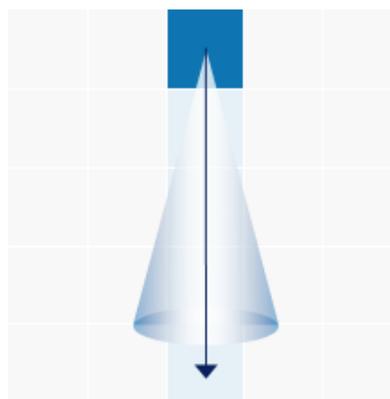


Figure 3: A TERMA voxel emitting a cone of energy. The only voxels that the dose is deposited to are along the collapsed ray line (arrow).

Forward vs. Inverted Calculations

It is easiest to visualize dose emanating from TERMA voxels and being deposited in the surrounding tissue. In a forward calculation, cones are cast outward in every direction spreading all of the energy released in that voxel (TERMA) to the surrounding voxels (dose). However, this is not a computationally efficient way to perform dose calculations. For example, performing one million forward calculations results in one million individual small doses that must be added on top of each other, which is difficult to parallelize (perform on many CPU or GPU cores simultaneously).

Thus, most CCCS calculations are “inverted”. In the inverted approach, cones are cast inward in every direction to calculate the energy received in that voxel (dose) coming from the energy released in surrounding voxels (TERMA). Neither variation is inherently more accurate, but the inverted method allows for an increase in the overall calculation speed.

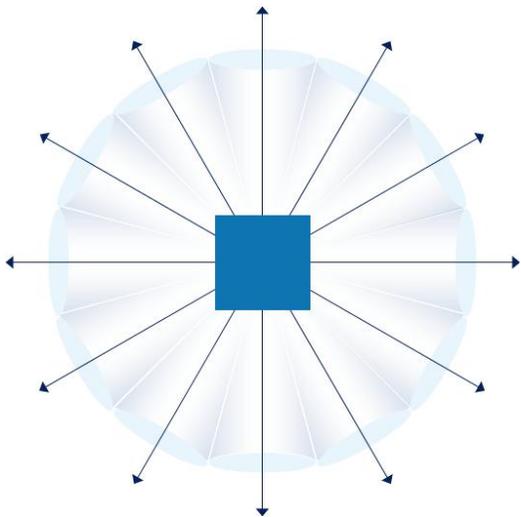


Figure 4: Forward calculation: Dose spreading from a TERMA voxel (cube) along cones in a forward manner.

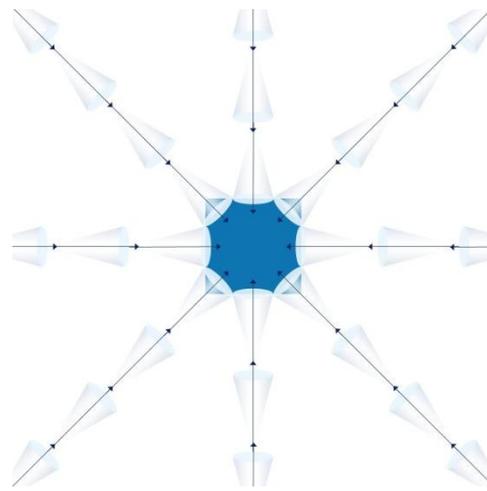


Figure 5: Inverted Calculation: Dose being collected in an inverted manner.

Mobius3D Collapsed Cone Model

Mobius3D Beam Modeling

Mobius3D comes with reference beam models for common linear accelerators. Users can choose to directly use a stock reference beam model, or enter key parameters (depth dose points, output factors, and off axis ratios) and have Mobius3D scale its reference beam data accordingly. After reference beam data is scaled to match your machine, the automodeling script will update Mobius3D’s beam model for the selected machine and energy combination.

Why Use Reference Beam Data?

- Time savings: By allowing you to enter a small subset of data for your machine, the beam model accurately reflects your machine’s output without the need for a lengthy commissioning process.
- Validates commissioning measurements: By using an independent set of measurements as the basis for your beam model, you are ensuring that errors in your original data

set do not propagate into your QA verification system.

Why Use Automodeling?

- Time savings: A few minutes entering basic machine information (vendor, model, MLC type, etc) along with a few key measurements from commissioning is all that is required to start building an accurate beam model. Then, in roughly an hour, automodeling will perform every step necessary to build the final model with no additional user interaction required.
- Dose calculation accuracy: A dose calculation algorithm is only as good as its beam model. If a model can be created that matches the input data (depth doses and profiles in a water tank), but does not perform well in clinical calculations (curved surfaces, heterogeneities, air gaps, etc.), the accuracy of the dose calculation suffers.

Automodeling allows Mobius3D to control and adjust every input parameter over a range of settings designed to match the dose calculation algorithm construction. This eliminates requiring physicists to master the intricacies of producing a great beam model in Mobius3D.

Mobius3D Fluence

After beam models are locked in, fluence calculations become the foundation for all dose calculations. They must be customized to each type of linear accelerator. Accurate fluence modeling is extremely important for IMRT and VMAT treatments, which are composed of many overlapping and abutting MLC segments. Any simplification of MLC motion during fluence calculations can have negative consequences in dose accuracy. Physicists

should be aware of how their TPS and TPS QA system model fluence in order to understand their resulting dose calculations.

Mobius3D's starting point for modeling fluence is a uniform fluence map - 100% intensity for unblocked areas of a beam and 0% intensity for blocked portions. Layers of adjustments are added to account for specialized properties of linear accelerators, such as curved MLC leaf ends, flattening filters, wedges, collimator and MLC transmission, etc.

Mobius3D's fluence map has a resolution of 1 mm. Partial-pixel effects are accounted for by reducing the intensity within the relevant fluence pixels (ie, a block at 50.5 mm is reduced to 50% fluence for the edge pixel).

Arbitrary Radial Intensity Profiles

Flattening filters are typically designed to produce a flat profile at a depth of 10 cm, but the incident fluence is not flat. Mobius3D models an arbitrary fluence across the width of the beam. The radial intensity profile most affects in-field profiles (i.e., off-axis ratios).

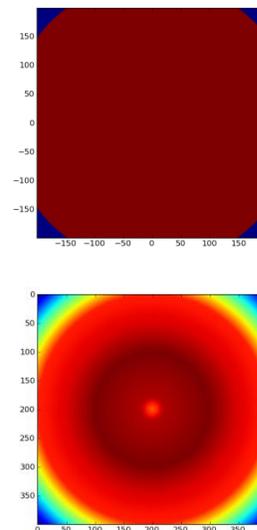


Figure 6: 2D fluence of the source through the primary collimator, before and after adding arbitrary fluence.

Gaussian source size correction

The photon source at the target is not a perfect point, but rather an area typically 1-2 mm in diameter. Mobius3D models the source as a Gaussian function to match the geometric distribution of the photon source. The Gaussian source most affects jaw and MLC penumbra calculations.

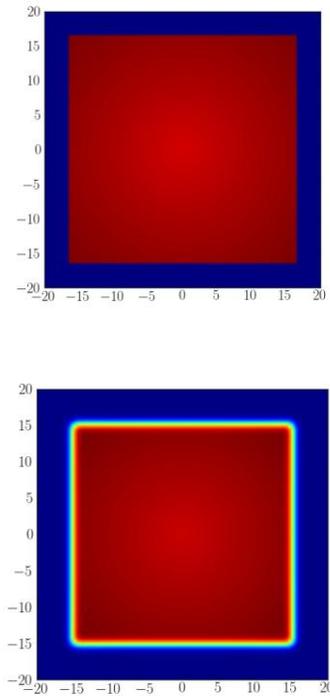


Figure 7: Fluence maps before and after applying the Gaussian source size correction.

Gaussian flattening filter scatter

The flattening filter is also a source of photons, due to photon interactions. Mobius3D models a secondary Gaussian source at the position of the flattening filter to account for this additional scatter. Gaussian flattening filter scatter most affects dose 1-3 cm outside of field edges, as well as output factors for small fields.

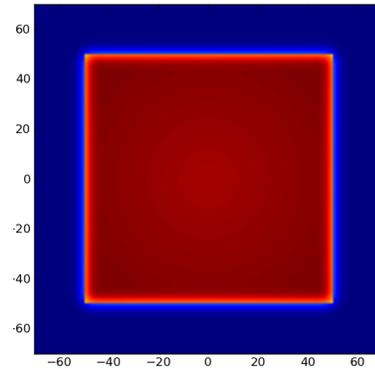
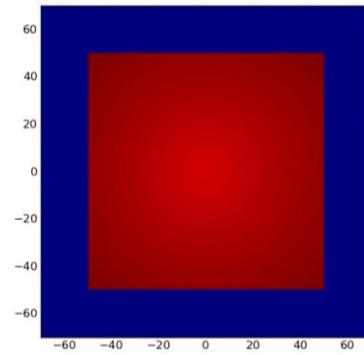


Figure 8: Fluence maps before and after adding Gaussian flattening filter scatter.

Field Collimation Cutouts

Main jaws and MLCs do not block 100% of the primary fluence, so Mobius3D models percent transmissions through each of these blocks.

Rounded MLC Leaf Modeling

Elekta and Varian MLCs use a tertiary MLC design that employs rounded leaf ends. The curvature of the leaf tips is added to account for the MLC's single-focused design (leaves move perpendicular to the gantry housing and do not tilt towards the source).

Mobius3D models Elekta and Varian leaf tips as circular. Fluence ray tracing is performed from the target and flattening filter sources through the curvature of the leaf tips to calculate the fluence impact along the leaf edge. Thus, the impact on the leaf's fluence profile is different depending on the leaf's position. Mobius3D also corrects for the light-radiation field offset

(more precisely, the difference between the 50% radiation edge and the MLC setting).

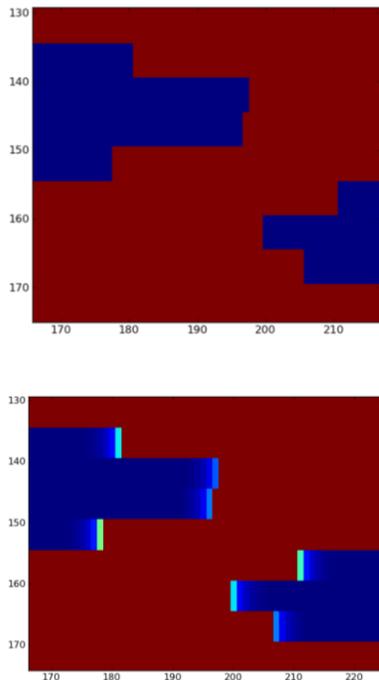


Figure 9: Fluence maps of MLC blocks (blue) before and after rounded leaf end corrections are applied, demonstrating that leaf position affects transmission.

Tongue and Groove

MLCs have a tongue and groove design to reduce the amount of interleaf leakage. Mobius3D accounts for this by making each leaf wider, and having the edge of each leaf block 50% as much radiation as the center. For example, a 5 mm MLC may be modeled as having a total width of 6 mm, but each 1 mm edge blocks 50% of the dose, and only the central 4 mm blocks the full amount (equal to the MLC transmission). The rounded leaf modeling is also carried into tongue and groove calculations.

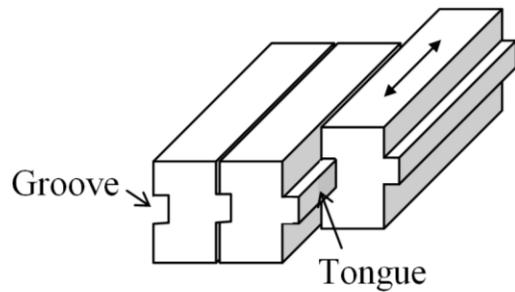


Figure 10: Illustration demonstrating tongue and groove usage on MLCs (illustration by Emma Viviers, posted at medphysfiles.com).

IMRT and VMAT summation

IMRT and VMAT plans contain MLC motion throughout the treatment. For step and shoot plans (containing several discrete MLC patterns, with the beam turned off between each one), Mobius3D calculates a full fluence for each pattern and sums the fluences weighted by monitor units.

In sliding window and VMAT, plans have leaf motion while the beam is on. For sliding window plans, Mobius3D calculates a full fluence when any leaf moves 1 mm from the last fluence calculation, and then sums these fluences weighted by monitor units to create a final fluence for the field. VMAT plans calculate one fluence map every 4 degrees of gantry motion. This fluence is calculated using a weighted average of all leaf motions that occurred within 2 degrees of gantry rotation from that point (the same as constructing a final fluence for sliding window fields), and not by simply using a snapshot of the current leaf positions at that gantry angle.

Mobius3D TERMA

After fluence is calculated, it is converted to several discrete energy bins (the photon spectrum determined in beam modeling) for TERMA calculation. Although fluence calculations ray trace from a Gaussian target

and a Gaussian flattening filter, TERMA calculations ray trace only from the photon target. NIST attenuation values are used to calculate the exponential decay in TERMA as the ray lines propagate through tissue.

The CT is converted to electron density via a user-defined CT to density table, along with any applicable overrides specified in a corresponding RT-STRUCT. It is assumed that the relative electron density equals the relative physical density (electron density applies to calculating radiological distances during ray tracing, and physical density is needed to calculate dose). Electron density is resampled to the dose grid resolution by trilinear interpolation. This ensures that accurate density for the dose-grid-size voxel is used in later computations.

Partial-volume Effects

The same voxel size is used for electron density, TERMA, and dose. It typically contains 1-3 mm wide voxels. Contributions to TERMA voxels from multiple points in the projected fluence map are supported by estimating their partial volume intersection with the TERMA voxel, allowing for arbitrarily fine fluence resolution.

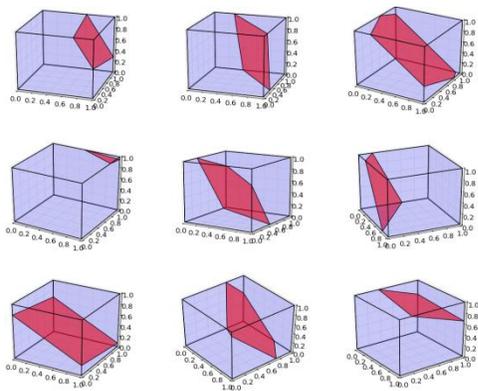


Figure 11: Sample intersections of voxels and planes, such as the edge of a projected fluence pixel.

Off-axis Softening

Flattening filters are thick in the center of the beam and taper off toward the edges of the beam. This means that the center of the beam is attenuated by more metal, and more low-energy photons are absorbed. Mobius3D

models this as:
$$\left[\frac{1}{1 + \frac{E_i}{E_{max}}} \right]^{S\theta}$$

Off-axis softening corrections are needed to match in-field profiles (off-axis ratios) at both shallow and deep depths.

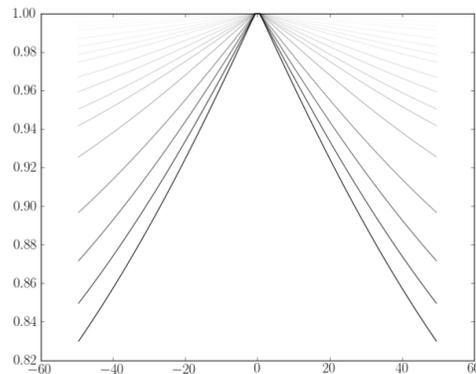


Figure 11: Radial dependence of different energy photons. Darker lines represent higher energy, normalized to their central axis value. The spectrum becomes softer at large radii (i.e., high energies are attenuated more rapidly).

Mobius3D Dose Deposition

Mobius3D uses an inverted, GPU-accelerated, collapsed cone calculation to deposit the TERMA as dose. 144 isotropically-spaced cones are used for each voxel, dose is accumulated at each voxel, and partial-volume effects are fully taken into account. Mobius3D's CCCS agrees with a full CS calculation within 1% inside the treatment field (including buildup regions) for a heterogeneous beam, with most areas agreeing within 0.3%.

Mobius3D uses high-resolution dose spread kernels calculated by Huang¹¹. These kernels were calculated using 6 times as many voxels as Mackie's original work³ with a more modern version of EGS that includes updated cross-sections and fewer assumptions.

Ray Tracing Accuracy

Each ray traces the exact linear distance through every voxel intersected by that ray. This correct distance, combined with the accurate electron density of the voxel ensures Mobius3D's computed radiological distance closely matches the true radiological distance. In addition, the Mobius3D ray tracing algorithm collects dose at every voxel. Using variable step sizes decreases calculation time, but can lead to inaccurate results while accumulating dose in regions with high TERMA gradients.

Inverted Calculation Assumptions

Mobius3D uses a careful treatment of the inverted CCCS algorithm to accurately distribute dose along collapsed cones. The inverted algorithm differs from a forward algorithm in that collapsed cone raylines originate from the dose deposition voxel rather than the initial interaction voxel. Using only the radiologic distance from an inverted calculation can lead to large dose errors near heterogeneous boundaries. Mobius3D attempts to match the energy-conserving forward algorithm by estimating what the intersection would be from a ray originating from the initial interaction voxel, which provides improved dose accuracy in heterogeneous regions.

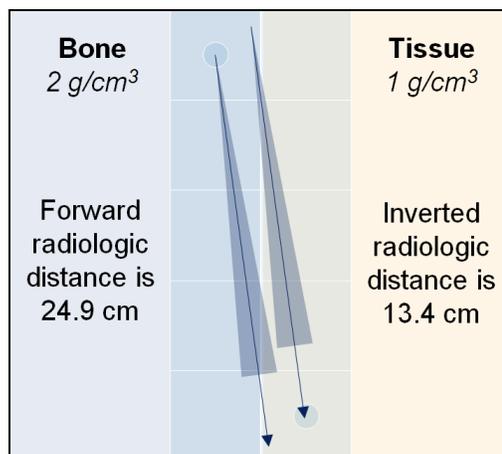


Figure 12: Example forward and inverted dose calculations for a cone near a bone/tissue interface.

Mobius3D GPU Acceleration

Graphical Processing Units (GPUs, or video cards) can perform calculations up to 200 times faster than CPUs, assuming the calculation can be split up into thousands of small problems that execute simultaneously. Therefore, CC calculations, which are primarily ray tracing problems, can be sped up tremendously by using GPUs.

Designing a new CC implementation to require a GPU allowed Mobius3D to make significantly fewer assumptions. For example, Mobius3D is able to emanate more cones from each TERMA point and perform ray trace calculations along every individual voxel in every cone. Traditional CC algorithms require sacrifices to be made in these parameters to achieve reasonable calculation speed.

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